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Another Look at Sine-Dwell Mode Testing

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A simple and systematic method is described for the measurement of natural modes of vibration through the application of single frequency sinusoidal forces using a minimum number of shakers. It is an explicit procedure which can be prescribed for execution by technical personnel with no previous mode test experience. Anticipated advantages in the application of this selective orthogonal excitation (SOREX) methodology include 1) the achievement of improved mode isolation, 2) elimination of the need for "many" shakers, 3) shortened test duration, and 4) freedom from dependence upon special insight of the test conductor. The method derives from a rejection of the classical use of phase coherence of response as a test criterion in favor of a more rational and practicable test logic.

Nomenclature

b	= scalar coefficient
f	= number of shakers
F	= vector of applied forces
g	= hysteretic damping coefficient
$[G]$	= generalized damping matrix
i	$=\sqrt{-1}$
$[I]$	= unit matrix
$[K]$	= generalized stiffness matrix
m	= number of modes
$[M]$	= mass matrix
n	= number of local coordinates
$q,\ddot{q}$	= generalized coordinate vectors
Q	= vector of generalized forces
$x,\ddot{x}$	= local coordinate vectors
γ	= orthogonality parameter, Eq. (11)
λ	= square of frequency ratio
Ψ	= contaminated mode vector, Eq. (12)
Ω',Ω''	= real, imaginary components of admittance, Eq. (9)
ω	= circular frequency

Superscripts and Subscripts

$[\ ]^T$	= matrix transpose
$[\ ]^{-1}$	= matrix inverse
$(\hat{\ })$	= truncated vector or matrix
$(\ )$	= generalized coordinate parameter
$(1),(2),(3)$	= iteration index
k	= selected coordinate
t	= target (resonant) generalized coordinate

Introduction

THE experimental measurement of natural modes of vibration is a continuing requirement for the accurate characterization of dynamic models of complex, lightly damped, nearly linear aerospace structures. For many years, the predominant procedure for the accomplishment of such measurements was the "sine-dwell" mode test. The logic of this technique involves the deployment of a number of shakers over the test article to exert sinusoidally varying forces at the natural frequency of a target mode, and then to adjust the position and relative magnitude of these forces so as to isolate the target mode response from all other modal responses. The mode parameters then are taken from direct observations of the forced vibration. Historically, this has generally been found to be an effective, although oftentimes difficult, lengthy, and therefore costly procedure, whose success has sometimes seemingly been dependent upon a special competency of the test conductor.

More recently, with the availability of special purpose analysis equipment, deduction of the parameters of the natural modes of vibration by digital processing of data from test responses to nonsinusoidal excitations has been receiving considerable attention and favor. The principal objectives are shorter testing time and elimination of the test conductors's "art" through largely automated data handling. For example, in the so-called *single point random* technique, samples of the responses to broadband random excitation of a single shaker are digitized and numerically processed into transfer functions. These "measured" transfer functions then are curve-fitted with a summation of simple oscillator characteristics, from which the mode parameters finally are deduced. In some instances, the method has succeeded admirably. However, in some instances involving complex articles with high modal density, the data processing task has been formidable and extremely time-consuming, and the ultimate results have not been completely satisfactory.

Although devoted advocates of each approach can be found to argue the relative advantages and disadvantages, the general superiority of one or the other of these approaches is not yet evident (see, for example, Leppert et al.,[1] and Stroud et al.[2]). Final resolution may well depend upon improvements of logic and/or equipment which have yet to be developed. Meanwhile, both methodologies leave ample room for improvement in practice, and each will undoubtedly see continued applications as special circumstances seem to merit.

In any event, it is not the intention of this paper to enter into this controversy: it is intended, rather, to suggest a variation of the multishaker sine procedure which is both more efficient and more effective than the traditional multishaker techniques. The potential advantages are such as to make the sine-dwell test much more attractive in time and cost, and much more explicit and certain of results than heretofore.

Traditional Test Methodology

The theoretical and practical considerations of performing a sine-dwell mode survey test have been investigated at length, and numerous erudite papers have been published in the technical literature over a period of several decades. In 1947, Kennedy and Pancu[3] described the characteristics of the real

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and imaginary vector components of vibration response with respect to the applied forces, and illustrated their utility in the conduct and interpretation of vibration testing. A significant feature of that treatment was the identification of phase coherence of response as a criterion for successful mode isolation. Lewis and Wrisley[4] in 1950, Traill-Nash[5] in 1958, Asher[6] in 1958, and Craig and Su[7] in 1974, among others, have proposed logics for the application of many shakers to achieve mode isolation using phase coherence as a criterion. These landmark papers provided much of the basis for the direction of the sine-dwell mode testing in these and more recent years.

Although some investigators (i.e., Stahle,[8] and Morosow and Ayre[9]) have suggested the use of post-test analysis procedures to separate modal data from the measured responses to simple nonselective sinusoidal excitations, such methods have not been widely adopted. Ibañez[10] asserted in 1976 that "Almost every large-scale modal test since 1950 has been performed using variations of the multiple-driver technique of Lewis and Wrisley." As described by Morosow and Ayre, in this approach, "...an iterative adjustment of force ratios and the excitation frequency is used to minimize the phase differences among various points on the structure and to bring all the displacement or acceleration measurements into quadrature with the excitation forces. It is basically a manual, iterative phase optimization process which is laborious and time consuming. Much depends upon the degree of skill of test personnel and their familiarity of the structure."

In the opinion of this writer, the traditional approach to the conduct of sine-dwell mode testing is fundamentally flawed as a practical procedure. Although phase coherence of response is, of course, an indication of a pure single-mode vibration, it is an unsuitable criterion for use in adjusting excitation forces to achieve mode isolation, or in evaluating the adequacy of an imperfect result. Phase coherence is virtually never achieved in practical test situations, where the number of available shakers is usually far exceeded by the number of modes, and where access to crucial areas of the test assembly often is prohibited. With typically complex test articles having as many as 30-50 modes in the frequency range of interest, with the number of available shakers limited to perhaps 6-12, and with the limitations of human skill and perseverance, it is not uncommon to arrive at a "best" excitation which is far from satisfactory. Clearly, a better criterion and a simpler and more workable methodology is needed.

Forced Vibration Response Characteristics

A sine-dwell mode survey is simply a measurement of the parameters of the forced vibration response, at the resonant frequency of each natural mode in turn, induced by phase-coherent sinusoidal excitation forces. The success of such a procedure depends upon finding an excitation which achieves an adequate degree of isolation and upon the choice of appropriately selective response parameters for measurement.

Let us re-examine the characteristics of forced sinusoidal vibration testing for a more practicable approach. For example, is the minimization of the response of each off-resonance mode equally important to the success of a mode measurement? What observables should be measured to most accurately secure the modal parameters? And, what logical procedure can be employed which will provide an explicit path to the needed mode isolation?

For this discussion, it is postulated that the test article of interest is a lightly damped, elastically linear structural continuum with an unlimited number of normal modes, and that its energy dissipation characteristics can be reasonably approximated by modal hysteretic damping. (We assert that this is adequately representative of most structures of interest.) It is presumed that the test objective is the measurement of the mode shape, frequency, and damping parameters of each of the natural modes in a specified frequency range.

In order to conveniently describe the nature of the proposed test vibrations, we follow conventional engineering practice and presume that the test article can be adequately approximated by a mathematical model with a finite number (n) of degrees of freedom. That is to say, the model is such as to have natural modes, in the frequency range of interest, which closely approximate the natural modes of the actual structural continuum.

In modal coordinates the equations of motion are written as

$$[\bar{M}]\ddot{q}+[\bar{K}]([I]+i[\bar{G}])q=Q \quad (1)$$

The modal coordinates q are defined by the transformation

$$x=[\Phi]q \quad (2)$$

and the modal forces are

$$Q=[\Phi]^T F \quad (3)$$

It is convenient to normalize the mode vectors of $[\Phi]$ to unit modal masses.
That is,

$$[\bar{M}]=[\Phi]^T[M][\Phi]=[I] \quad (4)$$

so that

$$[\bar{K}]=\text{diagonal array of } \omega_k^2 \quad (5)$$

$$[\bar{G}]=\text{diagonal array of } g_k \quad (6)$$

For a sinusoidal excitation at frequency ω, the shaker forces are written as

$$F=F_0 e^{i\omega t} \quad (7)$$

where F_0 is real. Letting $\lambda_k=(\omega/\omega_k)^2$, it follows that each of the (uncoupled) equations for modal acceleration response is of the form

$$\ddot{q}_k=Q_k\{-\lambda_k/(1-\lambda_k+ig_k)\} \quad (8)$$

$$=Q_k(\Omega'+\Omega'') \quad (9)$$

These are the familiar frequency response functions for single degree-of-freedom oscillators, as illustrated in Fig. 1, and this would be the character of the response ($\ddot{x}_k$) observable anywhere on a test article which is responding in a single pure

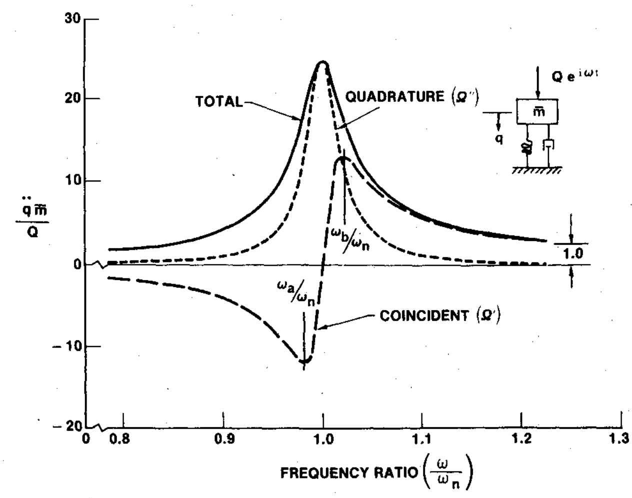

Fig. 1 Single-degree-of-freedom response spectra.

mode. The coincident component goes through zero, and the quadrature component maximizes, at $\lambda = 1.0$. Unfortunately, the real world seldom is this simple: invariably the observed response will include components of response from nonresonant modes.

As an example, Fig. 2 illustrates the superposition of the responses of several normal modes. Of course, the individual modal coordinate amplitudes are not observables in a test. What the test engineer monitors is the motion at each of a number of positions on the test article. Each of these is an (unknown) different linear combination of the normal mode coordinates. The $\ddot{x}$ parameter in Fig. 2 is illustrative of such observables. It is apparent that any attempt to measure the parameters of a normal mode from the forced vibration response, say mode 3 at ω_3, generally will include contaminating components from the off-resonance responses of all other modes. The quadrature component of response at resonance of the target mode is least contaminated by off-resonance mode contributions, since Ω'' diminishes more rapidly than does Ω' at frequencies remote from resonance. It provides approximations to natural frequency and mode amplitude which are significantly distorted only by nonresonant modes which are *close* in frequency and/or very large in relative amplitude. On the other hand, the amplitude of the coincident component of response is subject to troublesome amounts of contamination from responses of off-resonance modes that are quite remote in frequency (especially those that are lower in frequency), as well as from those that are close.

These considerations suggest that, when the vibration is potentially composed of contributions from many modes, the natural frequency of the resonant mode is most reliably approximated as the frequency at which the quadrature component of response maximizes, and the mode amplitude at each location is best approximated by the amplitude of the quadrature component of response at that frequency. The coincident component of response, and consequently the phase coherence, is more susceptible to off-resonance response contaminations and therefore should *not* be utilized. Clearly, if principal reliance is placed upon observations of quadrature response, the relative importance of contaminations from remote frequency modes is much reduced. It follows that phase coherence of response is an undesirable and inappropriate test criterion. With these observations, the stage is set for a somewhat heretical approach to sine-dwell testing.

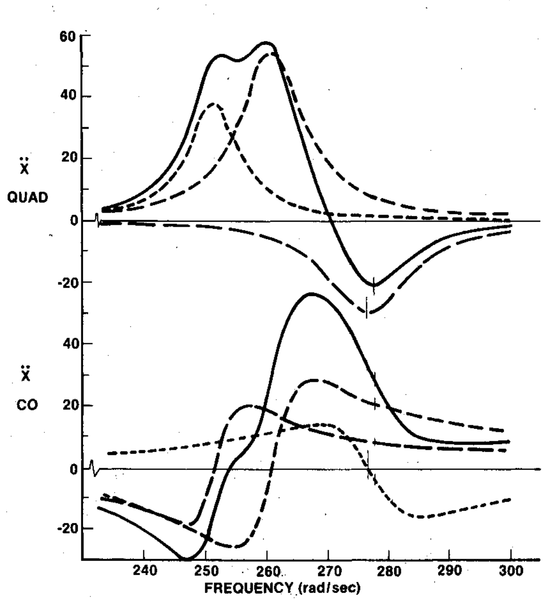

Fig. 2 Multimode response spectra.

A Simple Mode Isolation Logic

First, we need to select a parameter for measurement of the mode isolation that we require and to establish an acceptable bound for that parameter. Orthogonality between mode vector pairs seems to be an appropriate and workable parameter. The orthogonality γ of a measured mode pair is a scalar defined as

$$\gamma_{k,t} = \Phi_k^T M \Phi_t / (\Phi_k^T M \Phi_k \Phi_t^T M \Phi_t)^{1/2} \tag{10}$$

where Φ_k is the measured mode shape vector of any nonresonant mode and Φ_t is the measured mode shape vector of the target mode. When the mode shapes have been normalized to unit modal mass, this is more simply written as

$$\gamma = \Phi_k^T M \Phi_t \tag{11}$$

It is widely accepted in the mode testing community that an orthogonality parameter value of 0.10 or less is indicative of adequate mode isolation. (We expect to do better, but let us accept this value for discussion purposes.)

Now it is possible to quantify the tolerable contamination of the target mode vector Φ_t by a nonresonant mode vector Φ_k. The contaminated target mode vector can be written (neglecting mutual third-mode contributions) as

$$\Psi_t = b\Phi_k + \Phi_t \tag{12}$$

Orthogonality between Ψ_t and Φ_k is found to be

$$\gamma_{k,t} = b/(1+b^2)^{1/2}$$

$$\approx b \quad \text{for small } b \tag{13}$$

In other words, the orthogonality criteria state that the contaminating mode amplitude must be less than b times the target mode amplitude. (It is now understood that "mode amplitude" refers to the quadrature component of $\ddot{q}$.) Therefore

$$Q_k \Omega_k'' < b Q_t \Omega_t'' \tag{14a}$$

(For $b = 0.10$, this does not seem unduly restrictive.)

It is instructive to rearrange this expression

$$(\Omega_k''/\Omega_t'')(Q_k/Q_t) < b \tag{14b}$$

so that the sole condition for an adequate mode isolation is seen to consist of an inequality involving the product of two "selectivity" factors. The first factor is the ratio of quadrature admittances, the second factor is the ratio of modal forces. This inequality is the essence of the proposed *selective orthogonal excitation* (SOREX) logic.

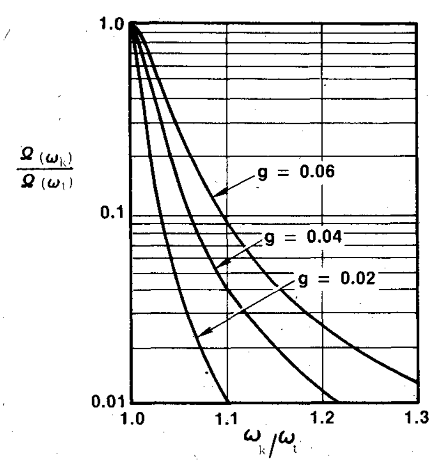

Fig. 3 Frequency selectivity of quadrature admittance.

Of course, the selectivity of the structural admittances Ω'' is fixed by the physical parameters of the test article. Figure 3 illustrates this selectivity for some typical structural damping ratios as a function of the frequency ratio ω_k/ω_t. It follows that an adequate mode isolation ($b<0.1$) would be achieved for all modes differing by more than about 10% from the target mode frequency, provided that the generalized force Q_k of the off-resonance mode is no greater than the generalized force Q_t of the target mode. This suggests that attention to the selectivity of the applied force vector F probably is necessary only for modes in close frequency proximity to the target mode, say within about 10-15%, depending upon the amount of damping present. (Since the number of such modes is relatively limited this should be a tractable problem; i.e., it should be practicable to achieve a very high degree of excitation selectivity for the limited number of modes.)

Next, the selectivity of a force vector F over a set of modes is quantified by the relative magnitudes of the corresponding generalized forces which result for each mode, Eq. (3). Recognizing that a small number (f) of shakers will be used, and that a limited number (m) of modes are to be considered, Eq. (3) can be reduced to

$$\underset{m\times m}{[Q_1 Q_2 \ldots Q_m]} = \underset{m\times f}{[\hat{\Phi}_1 \hat{\Phi}_2 \ldots \hat{\Phi}_m]^T} \underset{f\times m}{[\hat{F}_1 \hat{F}_2 \ldots \hat{F}_m]} \tag{15}$$

where $\hat{F}_k$ is a vector of forces for excitation at ω_k, $\hat{\Phi}_k$ is the kth mode vector (elements of shaker position coordinates only), and Q_k is the vector of generalized forces induced in each mode by F_k.

Perfect selectivity for a target mode at ω_t would be indicated by a finite value, say 1.0, for the generalized force of the target mode, and a zero value of generalized force for each other mode in the set. For example

$$[1 \;\; 0 \ldots 0]^T = [\hat{\Phi}_t \hat{\Phi}_2 \ldots \hat{\Phi}_m]^T \hat{F}_t \tag{16}$$

defines the force vector $\hat{F}_t$ of f elements which is capable of isolating the first mode of this group (at ω_t). This equation can be expanded to define similar force vectors for the isolation of each mode from the group in turn.

$$[I] = [\hat{\Phi}_1 \hat{\Phi}_2 \ldots \hat{\Phi}_m]^T [\hat{F}_1 \hat{F}_2 \ldots \hat{F}_m] \tag{17}$$

For $f=m$, an exact solution of Eq. (17) is obtained by inversion:

$$[F] = ([\hat{\Phi}]^T)^{-1} \tag{18}$$

For $f<m$, the "least-squared error" approximation is

$$[\hat{F}] = ([\hat{\Phi}][\hat{\Phi}]^T)^{-1}[\hat{\Phi}] \tag{19}$$

For $f>m$, a multiplicity of solutions exist, but these are of little practical interest in the present context.

Evidently, given correct values of the few mode shape vector elements $\hat{\Phi}_k$ for each mode, the desired excitation forces are readily calculable. But at this point, of course, the mode shapes are not known! This apparent shortcoming is remedied by our taking advantage of the natural selectivity of the structural filter which is the test article. As we have seen, the properties of the quadrature admittance are such that virtually any excitation at a natural frequency will tend to induce a large, if not dominant, response of the resonant mode. This suggests the use of an iterative approach. First, apply a simple excitation at each natural frequency in turn, and use the measured responses at the intended shaker locations as first approximations to the needed mode shape coefficients $\hat{\Phi}_k$. Then, calculate improved excitation forces, using Eqs. (18) or (19), and measure the responses to these excitations as improved $\hat{\Phi}_k$. (Conditioning of the $\hat{\Phi}$ matrix will be aided by the selection of shaker positions for which the signs of the quadrature responses form "orthogonal" arrays for the mode group. That is to say, a desirable shaker positioning is indicated if the columns of $\hat{\Phi}$, with all element magnitudes altered to unity, are independent.) The iterative procedure can be repeated until the generalized forces Q_k indicate that the desired forcing function selectivity has been achieved, or that covergence has occurred. For the $f=m$ case, convergence tends to be very rapid: typically one or two iterations should suffice (depending upon the crudity of the initial estimation) and convergence assures achievement of the intended isolation. (Conditions for convergence have been examined by W.P. Targoff in an unpublished memorandum.) However, for the $f<m$ case, we have specified an approximate solution and convergence may result in an unacceptable degree of isolation. Furthermore, for $f<m$, the degree of isolation achieved will depend upon the selection of shaker positions and upon the scale of the response vectors. Since this is a test characteristic which we have deliberately set out to eliminate, it is proposed that the $f<m$ option be avoided unless other practical constraints impose its adoption.

At this point the complete mode shape can be recorded. Ultimately, it will be checked for orthogonality against all other vectors of the measured mode set. (It is possible that unsatisfactory orthogonality may be found to exist between modes from different isolation groups. In this event, an examination of the generalized force matrix Eq. (15) will establish which mode or modes are faulty. The correction is obtained by adding the contaminating mode to the *isolation group* of the contaminated-mode and then calculating a revised force vector for isolation of the impure mode. No iteration should be necessary.)

Once the modes are reasonably isolated, damping is determined by performing narrow-band frequency sweeps around the natural frequency and measuring the frequencies of the characteristic maxima and minima of the coincident component of response (Fig. 1).

$$g = (\omega_a^2 - \omega_b^2)/(\omega_a^2 + \omega_b^2) \tag{20}$$

Even relatively large contamination of the coincident component of response by modes outside the isolation group will not significantly effect this computation of damping, since these contaminants are slowly varying in the vicinity of the resonant mode frequency. It is emphasized that damping measurement from free vibration decay records should not be employed, since the free decay vibrations can be expected to oftentimes contain large components of off-resonance modes. This is *not* to be taken as an indication of inadequate mode isolation.

A Test Procedure

The above discussion has outlined a logic from which a simple and explicit procedure for the conduct of a mode survey test can be extracted. The following is a brief summary of the method.

1) Obtain transfer function spectra for a number of positions on the test article sufficient to "identify" all the modes in the frequency range of interest and to include all potential shaker attachment positions.

2) Determine the natural frequencies and predominant motions of each mode.

3) For each target mode, select the group of modes sufficiently close in frequency to warrant inclusion in the determination of excitation forces (say, $0.85\omega_t \leq \omega_k \leq 1.15\,\omega_t$).

4) For each such *mode isolation group,* review the predominant responses, and select for shaker locations those coordinates with strong resonances and suitable phase reversals between modes. The number of shakers should be no greater than (preferably equal to) the number of modes.

5) Formulate first estimations of the truncated mode vectors $[\hat{\Phi}]^{(1)}$ by extracting the relative magnitudes of the

quadrature response acceleration peaks of the shaker position coordinates from the preliminary transfer functions.

6) Calculate improved force vectors $[\hat{F}]^{(1)}$ using Eqs. (18) or (19).

7) Apply the force vector $\hat{F}_t$ to the test article at each ω_t in turn, trim the frequency to maximize quadrature response, and record the quadrature response amplitudes $\hat{\Phi}^{(2)}$ at the shaker coordinates.

8) Recycle steps 6 and 7 until it is apparent that the relative magnitudes of the elements of $\hat{\Phi}$ have converged.

9) Measure and normalize the complete response vectors Φ_t, calculate the generalized force matrix by Eq. (15), and verify that adequate isolation has been achieved $[(Q_k/Q_t)<0.1$ for all $k \neq t]$. If not, recycle through steps 6-9.

10) Record the mode frequency as the frequency of maximum quadrature response. Record the mode shape from the maximum quadrature responses of each coordinate at the target mode frequency.

11) Perform narrow-band sweeps about the natural frequency, and record the frequencies at which the coincident response spectrum is maximum and minimum. Calculate damping, Eq. (20).

12) Repeat steps 5-10 for each target mode in turn.

13) Perform an orthogonality check of the complete mode vector set, and verify the isolation of the mode measurements. (Eliminate unacceptable intergroup contaminations, if any are found to exist, by expanding the isolation group of the contaminated mode to include the contaminating mode. Calculate a revised force vector and perform a remeasurement of the imperfect mode.)

Note that the test procedure has no dependence upon analytical representations of the test article. The mass matrix is used only in mode vector normalization and in the orthogonality check calculation which is performed after the iteration procedure is completed. All data are developed from experimental observations. No data processing, except phase resolution is required.

A Sample Test by Numerical Simulation

The method is illustrated below using a numerical simulation of a simple artificial test article. The example model has four modes, with properties as listed in Table 1. It is presumed that the test engineer has no prior knowledge of modal properties of the article; consequently, he begins his task arbitrarily by placing a single shaker at coordinate x_2. He performs slow sine sweeps and obtains the transfer functions shown in Fig. 4. From these data he concludes that there are four natural modes (with frequencies at approximately 251, 262, 277, and 411 rad/s). He observes that three modes are in fairly close proximity while the fourth is relatively remote. He decides to treat the first three modes an an *isolation group* using three shakers, and subsequently to measure the fourth mode with a single shaker.

For the three-mode group, he selects coordinates x_2, x_3, and x_4 as shaker positions, because these are responsive coordinates which exhibit suitable reversals of relative phase. He records the maximum quadrature component of responses $\ddot{x}_2$, $\ddot{x}_3$, and $\ddot{x}_4$ from the sine sweep response spectra at each estimated natural frequency; from these he assembles the truncated approximate mode matrix.

$$[\hat{\Phi}]^{(1)} = \begin{bmatrix} 46.0 & 45.0 & 24.0 \\ 51.0 & 54.0 & -20.0 \\ 60.0 & -34.0 & -7.0 \end{bmatrix} \tag{21}$$

and performs the calculation of force vectors, using Eq. (18),

$$[\hat{F}_l]^{(1)} = \begin{bmatrix} 5.14 & 4.09 & 24.15 \\ 2.43 & 8.55 & -20.70 \\ 10.66 & -10.41 & -0.92 \end{bmatrix} 10^{-3} \tag{22a}$$

The force vectors can, of course, be individually scaled as desired to conform to a preferred response level. Simply for convenience in this illustration, the force vectors have been scaled in a way which corresponds to the response vectors normalized to unit mass, i.e.,

$$[\hat{F}_l]^{(1)} = \begin{bmatrix} 0.155 & 0.110 & 0.417 \\ 0.072 & 0.231 & -0.357 \\ 0.320 & -0.280 & -0.016 \end{bmatrix} \text{(scaled)} \tag{22b}$$

These force vectors are then applied to the test article at each target mode frequency in turn, and responses at the shaker positions are recorded. That is, the forces $\hat{F}_1^{(1)}$ are applied and the excitation frequency is adjusted near ω_1 again to maximize the quadrature response. At this adjusted frequency, the quadrature responses of $\ddot{x}_2$, $\ddot{x}_3$, and $\ddot{x}_4$ are recorded for use as $\hat{\Phi}_1^{(2)}$. Next, the forces $\hat{F}_2^{(1)}$ are applied and the excitation frequency is adjusted near the estimated ω_2 to maximize quadrature response. At this adjusted frequency, the quadrature responses of $\ddot{x}_2$, $\ddot{x}_3$, and $\ddot{x}_4$ are recorded for use as $\hat{\Phi}_2^{(2)}$. A similar procedure is followed for $\hat{F}_3^{(1)}$ to obtain $\hat{\Phi}_3^{(2)}$. The three measured response vectors are assembled to form an improved $[\hat{\Phi}]$.

$$[\hat{\Phi}]^2 = \begin{bmatrix} 28.0 & 24.0 & 17.0 \\ 30.0 & 40.0 & -30.0 \\ 66.0 & -41.0 & -1.7 \end{bmatrix} \tag{23}$$

and a revised set of excitation force vectors is calculated by Eq. (18),

$$[\hat{F}]^{(2)} = \begin{bmatrix} 8.75 & 13.00 & 26.07 \\ 4.42 & 7.88 & -18.41 \\ 9.43 & -9.10 & -2.69 \end{bmatrix} 10^{-3} = \begin{bmatrix} 0.219 & 0.262 & 0.435 \\ 0.111 & 0.158 & -0.306 \\ 0.236 & -0.183 & -0.045 \end{bmatrix} \text{(scaled)} \tag{24}$$

These force vectors are used in a response measurement procedure as before with the result

$$[\hat{\Phi}]^{(3)} = \begin{bmatrix} 30.0 & 27.0 & 17.5 \\ 34.0 & 41.0 & -29.0 \\ 64.0 & -36.0 & -2.7 \end{bmatrix} \tag{25}$$

Table 1 Properties of example model[a]

Mode No.	1	2	3	4
Frequency, rad/s	251.3	260.8	276.5	410.3
Damping, g	0.04	0.05	0.06	0.07
Φ_{x1}	0.615	0.807	2.419	1.766
Φ_{x2}	1.164	1.336	1.018	−2.414
Φ_{x3}	1.307	2.040	−1.757	1.019
Φ_{x4}	2.560	−1.843	−0.147	0.152

[a] $[M]$ = diagonal array of $M_{ii} = 0.1$.

At this point it is expected that an adequate degree of isolation may have been accomplished. The iteration process is interrupted, the complete response vectors are measured and normalized, and the generalized forces are calculated.

$$[\Phi]^{(3)} = \begin{bmatrix} 16.0 & 16.0 & 40.0 \\ 30.0 & 27.0 & 17.5 \\ 34.0 & 41.0 & -29.0 \\ 64.0 & -36.0 & -2.7 \end{bmatrix}$$

$$= \begin{bmatrix} 0.63 & 0.80 & 2.41 \\ 1.19 & 1.36 & 1.05 \\ 1.34 & 2.06 & -1.75 \\ 2.53 & -1.81 & -0.16 \end{bmatrix} \text{(normalized)} \tag{26}$$

$$[Q]^{(3)} = [\hat{\Phi}^{(3)}]^T [\hat{F}]^{(2)}$$

$$= \begin{bmatrix} 1.00 & 0.08 & -0.01 \\ 0.08 & 1.00 & 0.05 \\ 0.00 & 0.03 & 1.00 \end{bmatrix} \text{(normalized)} \tag{27}$$

It is apparent that an adequate degree of isolation between the modes in this group has been accomplished and the complete response vectors are recorded as mode vectors. Narrow-band sweeps are performed for each mode to record the coincident response spectra for damping determinations, as shown in Fig. 5.

The convergence of the trial excitations is illustrated in Fig. 6, where the $\ddot{x}_2$ spectra for each of the three successive excitations are plotted for the first and second mode isolations.

Table 2 contains data that illustrate the progression of the iteration. The first row presents data for the single shaker excitations, the second row refers to the first three-shaker trial excitation, and the third row refers to the second three-shaker excitation. The first column contains the local force vectors, the second column consists of generalized force matrices of the applied forces, and the third column contains the orthogonality of the modes (as if they had been measured at each iteration stage).

Similarly, Table 3 illustrates the progression of an iteration using two shakers only, at coordinates x_3 and x_4. It is apparent that convergence leads to an imperfect isolation (as expected, since $f<m$) and, in this case, the partial isolation obtained does not meet the criteria for acceptability. Other two-shaker positions can of course be tried, but success is not assured. This illustrates the inherent limitations of the $f<m$ option and the fact that its use may be a false economy.

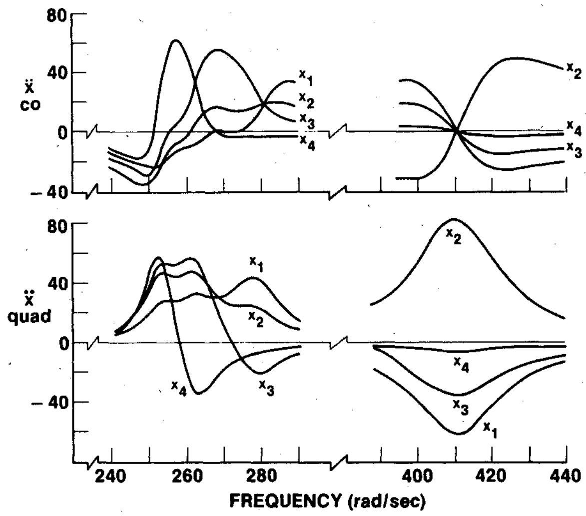

Fig. 4 Example response spectra for single shaker.

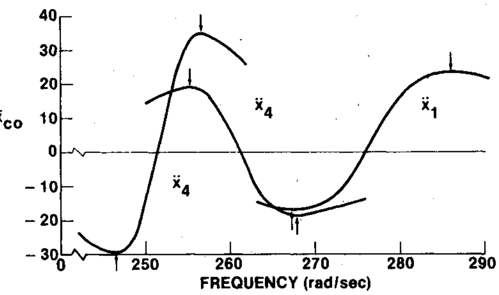

Fig. 5 Narrow-band coincident spectra from three-shaker end trials.

Table 2 Iterations of three-shaker excitation ($f=m$)

	$[\hat{F}]$			$[Q]$			$[\bar{M}]$		
	ω_1	ω_2	ω_3	ω_1	ω_2	ω_3	ω_1	ω_2	ω_3
1 shaker	1.00	1.00	1.00	1.00	0.92	1.10	1.00	0.46	0.16
(x2)	0	0	0	1.09	1.00	1.20	0.46	1.00	0.37
	0	0	0	0.91	0.83	1.00	0.16	0.37	1.00
3 shakers—	0.15	0.11	0.42	1.00	−0.36	−0.00	1.00	−0.13	0.01
first trial	0.07	0.23	−0.36	−0.24	1.00	−0.20	−0.13	1.00	−0.05
(x2,x3,x4)	0.32	−0.28	−0.02	−0.00	−0.20	1.00	0.01	−0.05	1.00
3 shakers—	0.22	0.26	0.44	1.00	0.08	−0.01	1.00	0.03	0.00
second trial	0.11	0.16	0.31	0.08	1.00	0.05	0.03	1.00	0.01
(x2,x3,x4)	0.24	−0.18	−0.05	0.00	0.03	1.00	0.00	0.01	1.00

Table 3 Iterations of two-shaker excitation ($f < m$)

	$[\hat{F}]$			$[Q]$			$[\tilde{M}]$		
	ω_1	ω_2	ω_3	ω_1	ω_2	ω_3	ω_1	ω_2	ω_3
1 shaker	1.0	1.00	1.00	1.00	0.92	1.10	1.00	0.46	−0.16
(x2)	0	0	0	1.09	1.00	1.20	0.46	1.00	0.37
	0	0	0	0.91	0.83	1.00	0.16	0.37	1.00
2 shakers—	0	0	0	1.00	−0.32	−1.15	1.00	−0.16	−0.01
first trial	0.15	0.29	−0.13	−0.26	1.00	−0.83	−0.16	1.00	−0.13
	0.31	−0.29	−0.04	−0.32	−0.42	1.00	−0.01	−0.13	1.00
2 shakers—	0	0	0	1.00	−0.23	−0.87	1.00	−0.10	−0.07
second trial	0.15	0.20	−0.21	−0.18	1.00	−1.05	−0.10	1.00	−0.25
	0.24	−0.16	−0.02	−0.36	−0.46	1.00	−0.07	−0.25	1.00
2 shakers—	0	0	0	1.00	−0.28	−0.82	1.00	−0.14	−0.04
third trial	0.14	0.18	−0.20	−0.22	1.00	−1.10	−0.14	1.00	−0.25
	0.25	−0.17	−0.01	−0.34	−0.43	1.00	−0.04	−0.25	1.00
2 shakers—	0	0	0	1.00	−0.29	−0.82			
fourth trial	0.15	0.18	−0.21	−0.20	1.00	−1.09			
	0.25	−0.17	−0.01	−0.35	−0.43	1.00			

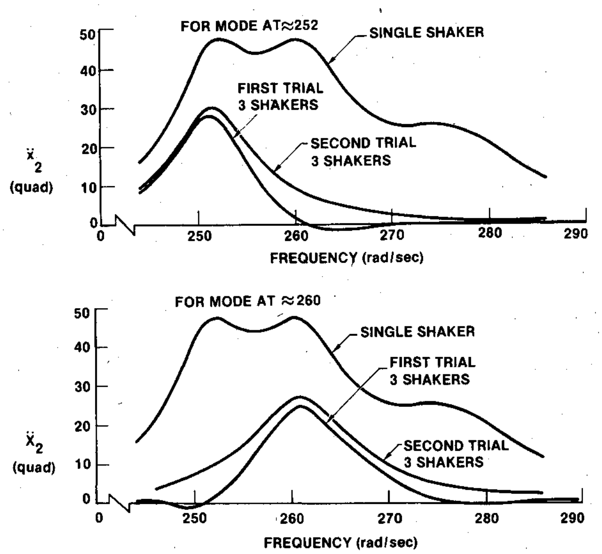

Fig. 6 Spectra for three-shaker iterations.

Conclusions

A critical examination has been made of the sine-dwell test procedure as generally practiced heretofore. It is concluded that the classical test criterion of a phase-coherent response at quadrature with the excitation forces is unnecessary, inappropriate, and should be discarded. Rejection of the phase coherence as a test objective avoids the imposition of unrealistic and unnecessary constraints and allows for the application of more rational test methodology. This methodology is explicit, rather than judgmental as before, and at the same time it is simpler and more effective.

A systematic and direct method, dubbed *selective orthogonal excitation* (SOREX), is described herein for the measurement of the natural modes of vibration through the application of single frequency sinusoidal forces using a minimum number of shakers. The number of shakers required for each mode measurement depends only upon the "density" of natural frequencies in the vicinity of the target mode frequency. Simple criteria are given for the selection of the shaker positions. The shaker relative force values are determined so as to minimize excitation of (only) those modes in close frequency proximity to the target mode. The method of force determination is iterative, successively involving the measurement of resonant response at each shaker coordinate and the calculation of improved force distributions. The required calculations involve products of matrices of order m (the number of modes in the isolation group) by f (the number of shakers applied) and the inversion of an f by f matrix. These manipulations are easily accomplished on site with a hand-held calculator. The complete response vectors need not be measured until it is desired to verify the selectivity of the resulting force distributions.

The motive for the development of the proposed methodology includes 1) the accomplishment of better mode measurements through improved mode isolation; 2) reduction of the elapsed time from test initiation to determination of final mode parameters; 3) an avoidance of need for "many" shakers; and 4) freedom from dependence upon special insight of the test conductor. It is asserted that the proposed method achieves all of these objectives, and that it does so without reliance upon complex data processing.

The procedure has been illustrated herein with reference to a simple lumped parameter model, using numerical simulations of the indicated experimental observations. It is recognized that this model, because of its simplicity, does not constitute a convincing demonstration of the efficacy of the procedure. However, experimental exercise of the method is not lacking, and the test experience is favorable. The subject logic has been used with success a number of times over a period of years to obtain improved mode measurements in instances where conventional multishaker sine test techniques were not adequately successful for all modes. It has been used also as a baseline method for the measurement of a complete set of modes of a current satellite configuration.

Test experience indicates a need for the ability to adjust phase between the several shaker force control signals to assure that a monophase force vector is delivered to the test article. (Because the proposed logic allows potentially large components of coincident phased response from remote frequency modes to occur, it is important that all applied forces be colinear, so that none of these responses will contaminate the nominal quadrature response measurements.) The principal problem derives from the inertial reaction of the armature mass, which at resonance will be 90 deg out of phase with the electromagnetic force exerted on the shaker armature. This phase control need not be closed-loop: a manual adjustment capability should suffice since measurements at fixed frequency are involved.

Acknowledgments

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